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ABSTRACT

The objective of this pretest-posttest Quasi-Experimental Design study was to determine the effects of LOGO Based Instruction (LBI) compared to instruction by teacher lecture and pencil-and-paper activities on: (1) students' understanding of the concepts of point, ray, line, and line segment; (2) students' attitudes toward learning geometry, learning LOGO, and learning through LBI; and (3) students achievment on a researcher-made test on the concepts of point, ray, line, and line segment. Sixty-seven students in the seventh and eighth grades of a Cincinnati middle school made up an experimental group (18 females and 17 males; 27 black and 8 white) that received LBI and a control group (13 females and 19 males; 24 black and 8 white) that received instruction by teacher lecture. Analysis of covariance indicated that students taught with a LBI program scored significantly higher (p .05) on the researcher-made posttest than the control group and showed significant differences (F=6.69-16.38, p .05) in students' positive attitudes towards math/geometry, LOGO, and LBI in favor of the experimental group. Tape recorded interviews of 32 students, 16 from each group, were analyzed to determine the conceptualization of the four geometry concepts. All students were on van Hiele Level 1 before treatment. After treatment, 14 students from the experimental group reached Level 3 and 1 student from the control group reached Level 2, indicating a significant difference (p .05) in favor of the experimental group in terms of gain in van Hiele levels. LBI for learning and teaching geometry at the secondary school level was found to be an effective method. (MDH)

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LOGO BASED INSTRUCTION IN GEOMETRY

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LOGO BASED INSTRUCTION IN GEOMETRY

The objective of the study was to determine whether secondary school students taught the geometry concepts of points, rays, lines, and line segments with Logo Based Instruction[1] (LBI) would score significantly higher; would have a more positive attitude towards learning geometry, learning Logo, learning through LBI; and would have better conceptualization of points, rays, lines, and line segments than students taught the same concepts by the teacher through lecture, and paper and pencil activities.

Secondary school students showed low performance in geometry at the national level (Carpenter, Corbitt, Kepner Jr., Lindquist, and Reys, 1981; Lindquist and Kouba, 1989). The achievement of students in American schools has become a matter of national concern, particularly achievement in science and mathematics (Carpenter, Matthew, Lindquist, and Silver, 1984). Lindquist and Kouba (1989) reported, from the results of the Fourth Mathematics Assessment of the National Assessment of Education Progress (NAEP) conducted in 1986, that there was a little improvement in the results of secondary school geometry over the previous national assessment. On the whole, secondary school students (7th and 11th graders) could not perform well in geometry. Students' performances were the best on the items that required the identification of common geometric figures. Their next best performance was on visualization tasks, followed by items requiring knowledge of geometric properties. The application items were the most difficult. Most of the students identified parallel lines, but only about one-third of the 7th grade students and two-thirds of the 11th grade students could identify perpendicular lines. The 7th grade students did not understand that squares are rectangles, that rectangles are parallelograms, and thus, squares are parallelograms. Fewer than half of the 11th-grade students were able to give the length of a diagonal of a square when given the area. It is important to note that only a little over half of those students who had taken geometry responded correctly to the item. Almost one-fourth of those students chose the "not enough information" option. In general, all secondary school students did not perform well on problems involving the Pythagorean Theorem. Thus, the national results in geometry indicated that secondary school students were weak in application of properties of geometric figures, construction of geometric figures, and problem solving.

The low performance in geometry reported above indicates possibly that the teaching of geometry at the secondary level could use new approaches. Perhaps many teachers are still teaching geometry by the traditional method, without using a computer as an aid to instruction. One approach to introducing new teaching strategies is to show teachers and students how to use computers as effective instructional tools for teaching and learning geometry. The National Council of Teachers of Mathematics (NCTM) devoted its 1984 yearbook to Computers in Mathematics Education emphasizing the importance of computers in mathematics education, advocated the use of computers in mathematics in Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and in Professional Standards for Teaching Mathematics (NCTM, 1991).

Difficult mathematical concepts could be made easy through computer simulation (Lappan & Winter, 1984). In fact, computers and computer graphics are new tools for teaching mathematical problems efficiently and effectively. Microcomputer software can be used as an in-class teaching aid creating three-way interactions among teacher, students, and microcomputers (Hativa, 1984).

In Logo programming, children reflect on how they might do the task themselves, and therefore on how they themselves think. The child programs to teach the computer how

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[1] Logo Based Instruction (abbreviated as LBI) is defined as using Logo computer language as a tool for instruction in addition to specified teaching methods and techniques.

to think, which helps the children to embark on an exploration about how they think. Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults (Papert, 1980). Logo computer language has been considered a natural choice for geometry activities (Battista and Clements, 1988a). There is a need to conduct research studies using Logo computer language as an instructional tool for teaching geometry at the secondary school level.

Methodology

This research study was designed to conduct a classroom experiment for teaching selected geometry concepts using LBI in one of the middle schools in Cincinnati in Spring 1990. A Quasi-Experimental Design (Campbell & Stanley, 1963) was used in order to find out the effects of teaching on students' achievement in geometry, students' attitudes, and students' conceptualization of abstract geometric ideas.

Hypotheses

Hypothesis 1

Secondary school students taught the geometry concepts of a point , a ray, a line, and a line segment with a Logo Based Instruction program would score significantly higher on the researcher-made test (Alpha = .05) than the students taught the same concepts by the teacher through lecture, and paper pencil activities.

Hypothesis 2

Secondary school students taught these geometry concepts with a Logo Based Instruction would have a more positive attitude towards learning geometry, learning Logo, and learning through LBI than the students experiencing a more traditional paper and pencil approach as measured by Likert Scales and Semantic Differential Scales.

Hypothesis 3

Secondary school students taught the geometry concepts of points , rays, lines, and line segments with a Logo Based Instruction program would have a better conceptualization of points, rays, lines, and line segments as compared with those in the control group.

Sample and Procedures of the Research Study

The sample consisted of 67 seventh and eighth graders. The students were divided into two groups, the experimental group (18 females and 17 males; 27 black and 8 white) and the control group (13 females and 19 males; 24 black and 8 white). Students in the experimental group were taught the basic turtle commands of Apple Logo II. They then were taught the concepts of points, rays, lines, and line segments through LBI, a set of tutorial modules written by the researcher in the Apple Logo II computer language. The control group was taught the same concepts using lecture, and paper and pencil activities. The regular classroom teacher taught both the experimental and control groups by following the treatment plans designed by the researcher. Taped recorded interviews of 32 students (16 from the experimental group and 16 from the control group) were analyzed to determine the conceptualization of the four geometry concepts. Two female teachers attended the experimental and control groups with their students. The teacher and the students were provided diskettes and instructions for completing the class activities and homework sheets. The researcher also developed the lesson plan for each topic for the experimental group providing teaching materials, class activities, individual activities, culminating activities, handouts, homework sheets, transparencies, review sheets, teaching techniques and methods of instruction.

The following instruments were developed and used to evaluate the effectiveness of LBI: (a) A pretest and a posttest were administered to evaluate students' achievement before and after the experiment, (b) Likert Scales and Semantic Differential Scales for the students and teachers were administered before and after the experiment to determine the attitude change towards Math/Geometry, Logo and LBI. (c) Interview sheets for the students were used to determine conceptualization of the four geometry concepts. Thought provoking questions about the figure, visibility, dimensionality, measurability, bisection of each of the four geometry concepts, and the possibility of cutting one concept by the other were asked. In addition, questions about the use of the computer for learning geometry, use of programming language and comments about the Logo Programming Exercises and Logo Tutorial Program were asked of both of the students and teachers involved in the experiment.

The instruments were validated by consulting three expert faculty members of the College of Education, University of Cincinnati, and two mathematics teachers. Reliability of the instruments was determined by the test and retest method. The instruments were administered in two classes of 44 ninth graders in one of the schools in the suburban area. The reliability coefficients for the pretest and posttest were 0.80 and 0.87 respectively.

Findings

Student Achievements

The Analysis of Covariance (ANCOVA) test, applied to pretest and posttest scores, showed significant differences at the 0.05 level ($F=30.35$, $p<0.05$) in the students' achievement for the four geometry concepts in favor of the experimental group. This result provided the evidence to accept the Hypothesis 1 that the secondary school students taught the geometry concepts of a point, a ray, a line, and a line segment with a Logo Based Instruction program would score significantly higher on the researcher made test (Alpha = .05) than the students taught the same concepts by the teacher through lecture, and paper and pencil activities. This was the combined effect for the use of Logo Programming Exercises, Logo Mathematics Tutorial 1 and the methods and techniques used under the LBI Treatment Plan (Yusuf, 1988, 1989, 1990). Particularly the flashing and movement of the figures on the computer screen helped the students to understand the four abstract concepts of geometry.

Conceptualization of a Point, a Ray, a Line, and a Line Segment

Properties of the Four Geometry Concepts

Figures

The students of the control group could not remember the importance of arrow heads on both sides of a line figure and on one side of a ray figure, while the students from the experimental group could remember the importance of both arrow heads on both sides of the figure of line, showing its continuity in both directions; and one arrow head on one side of the figure of a ray, showing its continuity in one direction. Therefore, most of the experiment group did not mix the figure of a line segment with the figure of a line. One reason for this is that their attention was gained through the flashing figures towards important points. They saw the flashing figures as they read the text on the computer screen. The flashing figures seemed to help students remember the ideas of the figures and

text. Moreover, they were asked to draw the figures on the computer screen using Logo commands. They had to pay attention to know how they could draw arrow heads on both sides of the figure of a line and on one side of the figure of a ray. This activity also helped their memory. Hence, the students performed better on the figures of lines and rays showing the concept of continuity using the arrow heads. After grasping the figures of a line and a ray, it was easier to remember the figure of a line segment (without arrow heads) and the figure of a point as a dot. In summary, visual presentation, flashing, and drawing of the figures on the computer screen by students using the Logo language commands seemed to help them remember the figures correctly.

Visibility

At the end of the experiment 25%, 31.25%, 12.5%, and 18.75% from the experimental group; and 0.00%, 0.00%, 0.00%, and 0.00% from the control group argued correctly against the visibility of a point, a ray, a line and a line segment, respectively. Hence, some experimental group students grasped the invisibility of the four geometry concepts better than the control group at the end of the experiment.

Most of the students from both of the groups before the experiment and from the control group only after the experiment were positive about the visibility of a point, a ray, a line and a line segment, having misconceptions about the figures of these geometry concepts as concepts themselves. Experimental group students had a better conceptualization of the invisibility of these abstract geometry concepts at the end of the experiment. However, many of the students from the experimental group and all the students from the control group thought of the figures of the four concepts and of the concepts themselves as concrete objects; therefore, they were positive about the visibility of the four geometry concepts. Invisibility of the four geometry concepts was the most confusing property for the students. The experimental group students performed better because they saw the figures flashing and read the text on the computer screen differentiating between the figures of the four geometry concepts and the abstract geometry concepts themselves.

Dimensionality

At the end of the experiment, 44%, 63%, 69% and 63% from the experimental group; and 0%, 0%, 6% and 0% from the control group responded correctly with correct reasons about the dimensions of a point, a ray, a line and a line segment, respectively.

Most of the students from the experimental group grasped the single dimensionality of a ray, a line and a line segment while only one student from the control group could grasp the single dimensionality of a line at the end of the experiment. A little more than half of the students from the experimental group and none from the control group understood the non-dimensionality of a point after the experiment. Hence, the experimental group students had a better conceptualization about the dimensionality of the four geometry concepts at the end of the experiment.

It was known from the try out phase that many students did not know the meaning of a dimension; hence, two examples were given before asking any questions about the dimensionality. Non-dimensionality of a point was a confusing property for the students. The experimental group students performed better than the control group students because the property was being repeated during the tutorial programs, class activities, homework sheets and review lessons.

Measurability

There were 68.75%, 43.75%, 62.50% and 62.50% students from the experimental group; and 31.25%, 6.25%, 0%, and 0% students from the control group who responded

correctly with correct reasons about the measurability of a point, a ray, a line and a line segment, respectively, at the end of the experiment.

After the experiment, most of the students from the experimental group understood the non-measurability of a point and a line and measurability of a line segment, and a little less than half of the students understood the non-measurability of a ray. Almost one-third of the control group understood non-measurability of a point and none understood non-measurability/measurability of the other three geometry concepts, except one student who understood the non-measurability of a ray after the experiment. It was concluded that the experimental group students had a better conceptualization about the measurability of the four geometry concepts at the end of the experiment.

The students of the experimental group were asked to draw figures of the four geometry concepts on the computer screen using the Logo computer language. Drawing Logo figures caused them to pay attention to the property of continuity. Continuity was represented by the arrow heads on one side for a ray and on both sides for a line; hence, students using Logo grasped the concept that a line could not be measured. They did not use any arrow in the figure of a line segment which provided them understanding that it could be measured. The non-dimensionality of a point was explained by using the position of the tip of the pencil, by flashing a dot on the computer screen and then by reviewing its property that it has no dimension. It was concluded that the Logo programs and teaching materials used for teaching the experimental group helped the students to conceptualize non-measurability/measurability of the four geometry concepts.

Bisection

After the experiment there were 75%, 50%, 62.5%, and 56.25% students from the experimental group; and 12.50%, 0.00%, 0.00%, and 18.75% students from the control group who responded correctly with correct reasons for the bisection of a point, a ray, a line and a line segment, respectively.

Fifty to 75% of the students from the experimental group and 0.0% to 18.75% of the students from the control group understood that a point, a ray and a line could not be bisected but a line segment could be bisected after the experiment. Therefore, it was concluded that the students of the experimental group had a better conceptualization of the bisection of the four geometry concepts at the end of the experiment.

Ability of One Geometry Concept to Bisect Another

The experimental group showed progress in understanding that only a line segment could be bisected by the other three geometry concepts (point, ray and line) after the experiment. On the whole, students from the experimental group conceptualized more clearly the interaction among the four geometry concepts with respect to their bisection of one another, than the control group students did at the end of the experiment.

From the experimental group 93.75% of the students, and from the control group 37.50% of the students responded correctly about the ability of one geometry concept to bisect another at the end of the experiment.

The students from the experimental group began to argue with reasons based on understanding after using the Logo Tutorials and Logo Programming Exercises because they were constantly being asked to provide reasons for their answers during the tutorial and exercises. Hence, they were able to better conceptualize the four abstract geometry concepts than the control group.

Student Attitudes

The ANCOVA tests, applied to the scores on Likert and Semantic Differential Scales, showed significant differences at the 0.05 level ($F=6.69\text{-}16.38$, $p<0.05$) in students' attitudes in favor of the experimental group showing a more positive attitude towards Math/Geometry, Logo and LBI. Hence Hypothesis 2 was accepted that the secondary school students taught the geometry concepts with Logo Based Instruction would have a more positive attitude towards learning the four geometric concepts, learning Logo, and learning through LBI than the students experiencing a more traditional paper and pencil approach as measured by Likert Scales and Semantic Differential Scales. In the beginning, the teacher explained to the experimental group that "Logo is an easy and powerful computer language for learning geometry." During the classes taught through LBI, they found that the Logo was helpful for learning the four abstract geometry concepts. The motivation and encouragement provided by the teacher and the power of Logo affected the attitudes of the students. Hence, their attitudes changed in a positive direction significantly as expected in Hypothesis 2.

Interview Sheet Scores

The significant difference found in favor of the experimental group at the 0.05 level ($F = 77.63$ $P < 0.05$) provided the evidence to accept Hypothesis 3, that the secondary school students taught geometry concepts of points, rays, lines, and line segments with Logo Based Instruction would have a better conceptualization of points, rays, lines and line segments as compared with those in the control group.

Van Hiele Levels

Mayberry (1981); Burger and Shaughnessy (1986); Bobango (1987); Crowley (1987); Olson, Kieren and Ludwig (1987); and Fuys, Geddes, and Tischler (1988) described the van Hiele model of the development of geometric thought. Different numbering systems for the levels of the model are found in the literature. The van Hieles referred to levels from the basic Level 0 to Level 4, others used Level 1 to Level 5. This study follows the Level 1 to Level 5 model. These levels are summarized as follows:

Level 1 (Basic Level): Visualization, recognition or appearance.

The student recognizes geometric figures by their shapes at this level. The student can learn geometric vocabulary. He/she can name, compare and operate on geometric figures (e.g., triangle, angles, intersecting or parallel lines according to their appearance).

Level 2: Analysis, description or properties of patterns.

Figures are recognized as having parts and are recognized by their parts. A student is able to compare shapes by these properties. Reasoning is now a part of the students' thinking. At this level a student analyzes figures in terms of their components and discovers properties/rules of class of shapes empirically (e.g., by folding, measuring, using a grid or diagram). However, relationships between properties cannot yet be explained by students, interrelationships between figures are still not seen and definitions are not understood.

Level 3: Informal deduction, theoretical, abstraction, order or interrelation of patterns.

At this level a student can establish the interrelationships of properties both within figures (e.g., in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and among figures (a square is a rectangle because it has all properties of a rectangle). A student can deduce properties of a figure and recognize classes of figures.

Level 4: Deduction or logical reasoning.

At this level, the significance of deduction as a way of establishing geometric theory within an axiomatic system is understood. The interrelationship and role of undefined terms, axioms, postulates, definitions, theorems, and proof is seen.

Level 5: Rigor or formal logic.

At this level a student can work in a variety of axiomatic systems; that is, non-Euclidean geometries can be studied and different systems can be compared. Geometry is seen in the abstract.

According to Hoffer (1981) students need to master large portions of the lower levels in order to function adequately at the more advanced levels on the van Hiele hierarchy. Battista and Clements (1988b) used Logo environments to help students progress through this hierarchy. They agreed that a proof-oriented geometry course requires thinking at van Hiele Level 4 but over 70 percent of the students begin high school geometry at Levels 1 or 2, and only those students who enter at Level 3 or higher have a good chance of becoming competent with proofs by the end of the course (Shaughnessy and Burger, 1985).

All students were at van Hiele Level 1 (Basic Level) before the experiment, while 14 students (87.5%) from the experiment group were at Level 3, and only one student (6.25%) from the control group was at Level 2 after the experiment. There was significant difference in favor of the experimental group at the 0.05 level ($t = 9.28$ $P < 0.05$) in terms of gain in van Hiele levels (see Table 1). This evidence also provided support for Hypothesis 3.

Student Interviews for LBI

Using a Computer and Computer Language

The majority of all the students were willing to learn geometry by using a computer. Most of the students from the experimental group liked to use the Logo computer language for learning geometry. They argued that Logo was easy and interesting and could teach them geometry. Therefore, Logo was considered by them a natural choice for learning geometry. Almost two-thirds from the experimental group, almost one-third from the control group and almost one-half of all the students preferred to use the computer alone, while 40% of all the students preferred to use a computer with a group of students. The students who preferred to work in groups at the computer argued that the group members could help each other to solve the problem, and one could talk to another member of the group. It is suggested that students be given the option to use the computer alone or with other students whenever possible.

The students of the experimental group commented positively about using the Logo Programming Exercises and Logo Tutorial Program. Their positive attitudes towards these Logo exercises produced a positive attitude towards learning and this enhanced their success at learning the four geometry concepts.

Table 1

Van Hiele Levels for the Geometry Concepts on the Interview Sheet

Experimental Group

S. #	Code #	Gender	Race	Grade	Level		Gain in Level
					Before	After	
1	1e	F	B	7	1	1	0
2	2e	F	B	7	1	1	0
3	3e	M	B	7	1	3	2
4	4e	F	B	7	1	3	2
5	5e	F	B	8	1	3	2
6	6e	M	B	8	1	3	2
7	7e	M	B	8	1	3	2
8	8e	M	B	7	1	3	2
9	9e	F	W	7	1	3	2
10	10e	F	B	7	1	3	2
11	11e	F	B	7	1	3	2
12	12e	M	W	7	1	3	2
13	13e	M	B	7	1	3	2
14	14e	F	W	7	1	3	2
15	15e	F	B	7	1	3	2
16	16e	F	W	7	1	3	2
				Mean	1.00	2.75	1.75
				S.D.	0.00	0.68	0.68

(continued on next page)

Table 1 continued

Control Group

S. #	Code #	Gender	Race	Grade	Level		Gain in Level
					Before	After	
1	17c	M	B	8	1	1	0
2	18c	F	B	8	1	1	0
3	19c	F	B	8	1	1	0
4	20c	M	B	8	1	1	0
5	21c	M	B	8	1	2	1
6	22c	M	B	7	1	1	0
7	23c	F	B	7	1	1	0
8	24c	M	W	7	1	1	0
9	25c	F	B	8	1	1	0
10	26c	F	B	8	1	1	0
11	27c	F	W	8	1	1	0
12	28c	M	B	8	1	1	0
13	29c	M	B	8	1	1	0
14	30c	M	W	8	1	1	0
15	31c	M	B	8	1	1	0
16	32c	M	B	8	1	1	0
				Mean	1.00	1.06	0.06
				S.D.	0.00	0.25	0.25

$t = 9.28$

$_{.95}t_{15} = 1.753, \; P < 0.05$

Note. Basic van Hiele Level has been taken as 1.

Teacher Interviews For LBI

All the teachers involved in the experiment liked to use a computer as an instructional tool for teaching geometry in their classes. Two-thirds liked to use Logo computer language, while one-third liked to use BASIC computer language for teaching geometry. The strongest reasons given in favor of Logo as an instructional tool for teaching geometry were: "(a) Logo is an easy computer language, and (b) It is a powerful computer language."

The teachers commented that the exercises and tutorial program were excellent, tied-in well with the lessons; flashing figures helped students to pay attention, and the option for calling the solution was appropriate. However, it was felt that more time was needed for teaching Logo. All the teachers thought that teaching methods and techniques used in LBI were useful and effective. They were of the opinion that the research experiment was a success.

None of the t values for teachers' scores on Likert Scales and Semantic Differential Scales was significant at 0.05% level (see Table 2). This is evidence that there was no significant change in the attitudes of the teachers towards Math/Geometry, Logo, and LBI at the end of the experiment. However, the range of the significant levels was from 0.16 to 0.42, which showed the teachers' attitude change in a positive direction.

Table 2
Teachers' Scores on the Likert and Differential Scales and t-Test Values

Code	Likert Scale						Differential Scale					
	Math/Geo		Logo		LBI		Math\Geo		Logo		LBI	
	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
T1	60	60	59	60	59	60	60	60	60	60	60	60
T2	56	60	36	58	36	53	55	60	36	44	36	52
T3	43	50	36	53	36	52	60	60	36	60	36	60
t values		1.81		2.11		2.19		1.00		1.51		1.89
Sig. at level		0.21		0.17		0.16		0.42		0.27		0.20

Note. $_{.95}t_2 = 2.92$ P > .05 NS*

Conclusion

The results of the ANCOVA tests, applied to the scores of achievement tests and interview sheets; and t-test applied to the gain in van Hiele levels, were significant in favor of the experimental group at the 0.05 level ($P < .05$). This means that these secondary school students taught the abstract geometry concept of points, rays, lines, and line segments with LBI scored significantly higher and had a better conceptualization of the four geometry concepts than those students in the control group (Hypotheses 1 and 3). The results of the ANCOVA tests applied to the scores on Likert Scales and Semantic Differential Scales were also significant in favor of the experimental group at the 0.05 level ($p < .05$) providing the evidence that secondary school students taught these geometry concepts with LBI had a more positive attitude towards learning geometry, learning Logo, and learning through LBI than the students experiencing a more traditional paper and pencil approach (Hypothesis 2). Thus, the three hypotheses were accepted. Hence, LBI for learning and teaching of geometry at the secondary school level was found to be an effective method. Further research needs to be conducted to verify the results of this research study.

Van Hiele Levels

It is clear from the research studies that identification of students' geometric thought levels are useful for revising the geometry curriculum and for planning geometry lessons. However, to identify a student's level of geometric thought is not an easy task for a classroom teacher because of the following two reasons: (a) Research findings showed that levels were not stable across concepts. A student may be at Level 2 for triangles and at Level 1 for congruency (Mayberry, 1981; and Burger, & Shaughnessy, 1986); (b) there is not any written standardized instrument which a teacher can use to find out the van Hiele level of each student for every geometric concept. A lot of research is needed to develop instruments to identify van Hiele levels of students for all geometric concepts in the curriculum. This study indicates that development of van Hiele levels for selected concepts is possible.

Comprehensive Logo Research Project (CLRP)

On the basis of the review of the research studies in the field and the positive results of my study, I would like to suggest a Comprehensive Logo Research Project (CLRP) addressing the all important issues described else where with large and representative samples from elementary, secondary and teachers' college levels, for resolving the issues (Yusuf, 1990, pp 56-59). On the basis of the results of the CLRP, recommendations for integration of the use of Logo programming in the mathematics curriculum should be made. In this way we will be able to enhance students' performance in geometry in particular and in mathematics in general.

NCTM should develop a CLRP involving all the experts in the field, prominent Logo researchers, representatives of elementary and secondary school, math teachers, and concerned instructors at teachers' colleges. The involvement of all the concerned personnel in the CLRP would facilitate the implementation of the recommendations. Each person who would have contributed to the project would like to see the recommendations implemented in the real world. Moreover, due to involvement of persons from every level, each level would cooperate whole-heartily, at every stage of the proposed Comprehensive Logo Research Project. The use of Logo programming should be integrated in the geometry curriculum on the recommendations of the CLRP.

Logo Based Instruction

Logo Based Instruction can be used to improve student achievement on points, rays, lines and line segments and for a better conceptualization of these abstract geometry concepts (for gain in van Hiele levels) at the secondary school level. It also can be used to create a positive change in students' attitudes towards Math/Geometry, Logo and LBI. Since students differ in their preference to work alone or with each other, it might be better if the students are given the option to use the computer alone or with one or more other students. Students should be taught Logo for at least two weeks before teaching them through Logo Based Instruction, then they would learn more from LBI tutorials. All the Logo programs and exercises should be easy and tied-in well with the lessons.

The results indicated that the students in the experimental group had a deeper conceptualization of the four geometry concepts after the LBI. The results also indicated that the students in the experimental group had significant increases in their van Hiele levels in comparison to the students in the control group. These results are encouraging and support the use of Logo Based Instruction in teaching geometry at the secondary school level. These basic abstract geometry concepts are important for developing a sound foundation for further knowledge of geometry. It is recommended that the research study should be replicated and extended to different situations to verify the effects of Logo Based Instruction in geometry. If the future research studies would confirm the effectiveness of LBI for teaching and learning geometry then LBI could be integrated into the geometry curriculum.

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